Neutrino Mass Models

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Neutrino Masses

- LEP experiments have shown us that there are only three light active flavours. I will assume that these are the only light neutrinos.
- These three flavours mix to form three mass eigenstates. We label them ν_1, ν_2 and ν_3 with masses m_1, m_2 and m_3 respectively.
- We need two independent mass-squared differences, $\Delta m^2_{\rm sol}$ and $\Delta m^2_{\rm atm}$, to account for the neutrino oscillation data.
- Without loss of generality, we set

$$\begin{aligned}\Delta m^2_{\rm sol} &= m_2^2 - m_1^2 = \Delta m^2_{21} = 7.5 \times 10^{-5}\ {\rm eV}^2 \\ \Delta m^2_{\rm atm} &= m_3^2 - m_1^2 = \Delta m^2_{31} = 2.5 \times 10^{-3}\ {\rm eV}^2.\end{aligned}$$

Note that $m_3^2 - m_2^2 = \Delta m^2_{32} \approx \Delta m^2_{31}$.

Neutrino Masses

- We have no idea what the neutrino masses actually are. We do have these two upper bounds:

$$\begin{aligned} m_{\beta\mathrm{kin}} &\leq 2\ \mathrm{eV}, \\ m_{\beta\beta 0\nu} &\leq 0.5\ \mathrm{eV}. \end{aligned}$$

- If we do not know what the neutrino masses are, then what method do I use to label them as ν_1/ν_2 etc?
- We use the data that is at hand. What is measured is the overlap between the electron neutrino and each of the three mass eigenstates.
- We use $|U_{ei}|^2$ to label the mass eigenstates. The mass eigenstate with the largest overlap is labelled ν_1 and the one with the smallest overlap is labelled ν_3.
- Given these definitions, Δm^2_{ij} can be positive or negative. Solar neutrino data requires Δm^2_{21} to be positive.

Neutrino Masses

- Sign of Δm^2_{31} is not determined yet. We use the labels Normal Hierarchy (NH) to denote positive value and Inverted Hierarchy (IH) to denote negative value.
- For NH (IH), m_1 (m_3) is the lightest mass. It can be unmeasurably small or it can be as large as the experimental upper bound. Given the lowest mass, the other two masses are determined by the mass-squared differences.
- Whatever the actual situation, neutrino masses have to be smaller than 2 eV, a million times lighter than the next lightest particle, electron.
- The question we face is: How to theoretically describe such a small mass?

Most boring way to generate Neutrino Mass

- In the Standard Model, fermion masses are generated by their interaction with the Higgs field,

$$\mathcal{L}_{\rm Yukawa} = Y^d_{ij}\bar{Q}_{iL}\Phi d_{jR} + Y^u_{ij}\bar{Q}_{iL}\tilde{\Phi} u_{jR} + Y^e_{ij}\bar{L}_{iL}\Phi e_{jR} + h.c.$$

- These terms generate both electron mass ($Y^e v/\sqrt{2} = 0.5$ MeV) and top quark mass ($Y^t v/\sqrt{2} \approx 200$ GeV). The top quark Yukawa coupling Y^t is about 1, so the electron Yukawa coupling is fine-tuned to be smaller by a factor of a million or so.
- If we add three right-handed neutrinos to our model, then we can add a fourth term to the Yuakawa Lagrangian

$$Y^\nu_{ij}\bar{L}_{iL}\tilde{\Phi}\nu_{jR} + h.c.$$

  By choosing $Y^\nu \leq 10^{-12}$, we can get neutrino masses of an eV or less.
- We worsen the fine-tuning in Yukawa couplings from one in a million to one in a trillion!

Majorana Mass of Neutrino

- The above mechanism is hated by theorists because (a) it contains no new idea and (b) it contains extreme fine tunning. It is also hated by experimentalists because it predicts that the only signature of neutrino masses is neutrino oscillations. In particular, it predicts that flavour violation is charged lepton sector is $\leq 10^{-50}$.
- But this is a viable model, which accounts for all the current data.
- The more popular method to generate neutrino masses is the See-Saw mechanism which gives rise to Majorana masses for neutrinos.
- As we saw from the Yukawa Lagrangian, a fermion mass term couples a left-chiral spinor to a right-chiral spinor.
- Given a left-chiral spinor ν_L, we can construct a right-chiral spinor $(\nu_L)^c \equiv C(\bar{\nu})^T$, where C is the charge conjugation matrix $i\gamma^2\gamma^0$.
- Similarly given a right-chiral spinor ν_R, we can construct a left-chiral spinor $(\nu_R)^c = C(\bar{\nu})^T$.

Majorana Mass of Neutrino

- In addition to the Dirac mass term $m_D(\bar{\nu}_L \nu_R + h.c.)$, we can also write Majorana mass terms

$$m_L\left(\bar{\nu}_L (\nu_L)^c\right) + m_R\left(\overline{(\nu_R)^c}\nu_R\right) + h.c.$$

  1. Majorana mass terms violate all additive quantum numbers by two units.
  2. Charge conservation forbids Majorana masses for charged fermions.
  3. Majorana masses for neutrinos violate lepton number by two units. They lead to neutrinoless double beta decay $N(A,Z) \rightarrow N'(A,Z+2) + e^- + e^-$.

- Majorana mass for left-chiral neutrinos m_L is forbidden in SM because $\bar{\nu}_L(\nu_L)^c$ has weak isospin $T_3 = -1$. SM has no scalar particle with weak isospin $T_3 = 1$ which can couple to it.

See-Saw Model

- Right-chiral neutrinos have no weak isospin and no hypercharge. So one can just write a Majorana mass term m_R for them in the Lagrangian.
- Since both left-chiral and right-chiral neutrinos are there, we can also generate a Dirac mass term through coupling them both to the Higgs doublet (as discussed earlier).
- In See-Saw model, we include both the Dirac mass m_D and the Majorana mass m_R. The mass terms can be arranged as

$$\left[\begin{array}{cc} \bar{\nu}_L & \overline{(\nu_R)^c} \end{array}\right] \left[\begin{array}{cc} 0 & m_D \\ m_D & m_R \end{array}\right] \left[\begin{array}{c} (\nu_L)^c \\ \nu_R \end{array}\right].$$

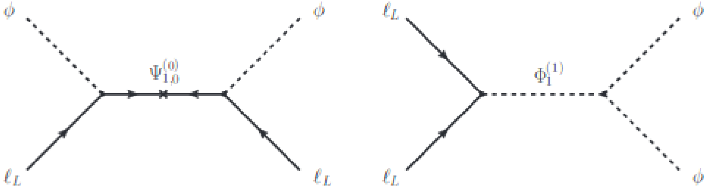
By diagonalizing the matrix, we get two eigenvalues, m_D^2/m_R and m_R.

See-Saw Model

- If we choose m_D to be of the order of electron mass and $m_R \sim 10^3\ \mathrm{GeV}$, we get a light neutrino mass of 1 eV (and a heavy neutrino of mass 10^3 GeV).
- In see-saw models, we avoid the extreme fine-tuning of 10^{-12} and return the usual fine-tuning of 10^{-6} of the SM.
- As mentioned earlier, Majorana mass leads to neutrinoless double beta decay.
- If we introduce CP-violation along with Majorana mass, we can also solve the problem of matter/anti-matter asymmetry.
- The lepton number violating decays of heavy Majorana neutrinos, along with CP-violation, can create a large ΔL (leptogenesis). Various mechanisms are proposed to convert this ΔL to ΔB (baryon asymmetry).

Neutrino Mass through Interactions

- So far we have discussed generating neutrino mass from the Lagrangian directly. It is also possible to generate neutrino mass through interactions.
- Consider the Feynman diagrams below



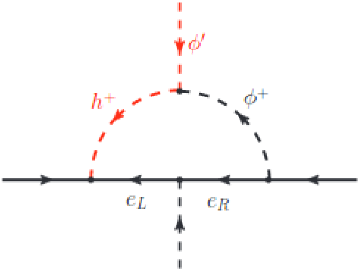
If the mass of the exchange particle is very large, it leads to an effective dimension-5 operator, called Weinberg operator.

$$\mathcal{L} = \frac{\lambda}{M}\left(\overline{\tilde{l}_L}\Phi\right)\left(\tilde{\Phi}^{\dagger} l_L\right),$$

where $\tilde{l}_L = i\sigma_2 (l_L)^c$, $\tilde{\Phi} = i\sigma_2 \Phi^*$, M is heavy mass of exchange particle.

Neutrino Mass through Interactions

- On electroweak symmetry breaking, this leads to a Majorana mass of neutrinos $m_\nu \sim \lambda v^2/M$.
- We get sub-eV neutrino mass for $M \sim 10^{15}$ GeV (GUT scale), if $\lambda \sim 1$.
- In the above case, we generated neutrino mass through tree level interactions of the theory from higher energy scale.
- There are a number of models (Zee model) where neutrino mass is generated through loop-level interactions of a higher energy theory.



Symmetries of Neutrino Mass Matrix

- The neutrino flavour eigenstates are related to the mass eigenstates by a unitary mixing matrix called PMNS matrix.
- This matrix is the diagonalizing matrix of the light neutrino mass matrix which is a complex symmetric matrix.
- PMNS matrix is parametrized by three angles and a phase as

$$U = R_{23}(\theta_{23}) U_{13}(\theta_{13}, \delta) R_{12}(\theta_{12}).$$

- The measured values of these angles are $\sin^2\theta_{13} = 0.02$, $\sin^2\theta_{12} = 1/3$ and $\sin^2\theta_{23} = 1/2$. No measurement of δ yet.
- Various efforts are made to constrain the neutrino mass matrix through discrete symmetries so that the above values are realized through the symmetries.

$\mu \leftrightarrow \tau$ Symmetry

- The exchange $\mu \leftrightarrow \tau$ symmetry is the simplest symmetry we can consider for neutrino mass matrix.
- The mass matrix takes the form

$$\begin{bmatrix} a & b & b \\ b & c & d \\ b & d & c \end{bmatrix}.$$

- It is straight-forward to show that the diagonalizing matrix has $\theta_{23} = \pi/4$ and $\theta_{13} = 0$.
- To obtain viable value of θ_{12}, we need to fine-tune $(c + d - a) = b$.
- Also the symmetry between 12 and 13 elements has to be badly broken to obtain even the small value of $\sin^2 \theta_{13} = 0.02$.

A_4 Symmetry

- This is the group of even permutations of four objects. It is the most popular discrete group in neutrino mass models. It is the smallest group which allows a triplet representation.
- There are a huge number of models based on this symmetry group.
- The model of He, Keum and Volkas (hep-ph/0601001) leads to Tri-bimaximal form of PMNS matrix purely from the symmetry without any fine-tuning.
- That is, the model predicts $\sin^2\theta_{12} = 1/3$, $\sin^2\theta_{23} = 1/2$ and $\sin^2\theta_{13} = 0$. This prediction is independent of the values of neutrino masses.
- To obtain non-zero θ_{13} a further modification of the model is needed.
- We showed (arXiv:1504.04034) that a perturbation invariant under $Z_2 \times Z_2$ (a sub-group of A_4) to the neutrino mass matrix leads to appropriate value of $\sin^2\theta_{13}$ and also predicts maximal CP violating phase.

Conclusions

- The question of how neutrino masses and mixing are generated is wide open.
- A more fundamental question is: Are neutrinos Majorana Fermions?
- If we see a signal for neutrinoless double beta decay, the answer is YES.
- A model where neutrinos have only Dirac masses through Standard Model Higgs is acceptable at the moment.
- There is an overwhelming preference for Majorana masses for neutrinos because that leads to other interesting phenomena such as observable charged lepton flavour violation and leptogenesis.
- We await for more data on (a) neutrino mass scale, (b) neutrino mass hierarchy, (c) CP-phase and (d) charged lepton flavour violation to discriminate between different neutrino mass models.